# BIOLOGICAL TEMPERATURE RETRIEVAL BY SCANNING RADIOMETRY

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## ABSTRACT

This paper refers to the retrieval of two-dimensional thermal fields in biological bodies from multispectral microwave measurements by a scanning waveguide applicator. A formulation of the inverse radiometric problem suitable for a two-step solution (transverse deconvolution and profile retrieval) is outlined. The performance of the inversion procedure is tested by a numerical evaluation of the impulse response of the resulting algorithm.

## INTRODUCTION

In hyperthermia treatment of loco-regional tumors the thermal field is needed both over the entire diseased region and in the adjacent normal tissues. Implanted sensors, such as thermocouples or optical devices are used currently to obtain data points from which the temperature field can be extrapolated by a suitable thermal modelling. Noninvasive remote sensing techniques have been proposed to avoid the uncertainties inherent in this procedure as well as to limit the discomfort to the patient. Among these, microwave radiometry is under consideration for mapping the temperature pattern in tissues as is required by the monitoring and control procedures in clinical hyperthermia protocols. Some experimental validation of the technique has been carried out by single-channel systems using additional information and auxiliary thermal data [1]. Multispectral techniques have been suggested which can retrieve the temperature patterns from measurements of the radiation emitted in different microwave frequency bands [2,3]. The efficiency of the technique in retrieving one-dimensional temperature distribution from noisy data has been discussed through numerical simulations [3-5]. More recently, the feasibility of the radiometric retrieval of two-dimensional temperature patterns has been analysed on the basis of the over-all pulse response [6]. The main conclusions were that, at least in principle, spatial resolution can be achieved both in a transverse direction and in depth. But enhanced processing difficulties were encountered deriving essentially from the ill-posed nature of the problem. This effect results in a dramatic loss of accuracy of the temperature reconstructions when realistic noise is introduced into the radiometric data.

To circumvent the difficulty, a formulation of the inverse problem has been worked out which leads to a two-step procedure: a transverse deconvolution first, and a profile retrieval thereafter. This technique improves the conditioning of the problem and leads to an algorithm which, on one side, is more robust with respect to noise, and, on the other, can be more easily implemented on commonly available computing facilities. The reported numerical simulation is mainly intended to characterize the spatial discrimination capability of the technique by analysing the pulse response of the resulting inversion algorithm. Some practical aspects of the use of a scanning waveguide contacting applicator are also included in the simulation.

## METHOD

We shall consider the thermal emission from a half-space and denote the transverse and longitudinal coordinates by $x$ and $z$, respectively (Fig. 1). Both the complex dielectric constant and the temperature are function of position for $z < 0$. The microwave radiometric sensor (truncated waveguide, horn) is placed in the half-space $z > 0$ in contact with the body surface. Both the structure and the sensor are invariant with $y$. For sake of simplicity we assume that power is exchanged between the receiving device and the body only through the aperture on the front-end of the sensor contacting the body. The dielectric filling the waveguide or the horn is assumed lossless.

It can be shown that the output of a monochromatic receiving device, at frequency $\omega$ is related to the physical temperature $T(x',z')$ inside the tissues by the integral equation

$$v(\omega,x) = c_1 \iint_{z'<0} \tilde{W}(\omega,x,x',z')\ T(x',z')\ dx'\ dz' + c_2 \tag{1}$$

where $\tilde{W}$ is a weighting function which represents the contribution by an elementary cell centered at $x',z'$, and $x$ denotes the position of the sensor (e.g. the middle-point of the aperture). Constants $c_1$ and $c_2$ take into account the amplification and the off-set of the receiver respectively. In Eq. (1) the temperature is measured with reference to an arbitrary reference value, i.e. the reference termination temperature used for calibration. As a consequence of Lorentz reciprocity in antenna theory, $\tilde{W}(\omega,x,x',z')$ can be obtained (after suitable scaling) as the distribution of power absorbed by cell $dx'\,dz'$ when the radiometric sensor is placed at $x$ and used as an active device radiating onto the body. In the following we shall assume $W=c_1\tilde{W}$ and $c_2$ in such a way that the unitary increase of $T$ above the reference temperature induces the unitary variation of output $v$ above the zero value, when $T$ is uniform throughout the body.

The inverse problem is the retrieval of $T(x',z')$ from measured values of $v(\omega,x)$, when the position $x$ of the sensor is moved across the surface of the body within the interval $|x| \leq A/2$, where $A$ is the width of the region to be explored. By assuming the temperature outside such a region to be zero, by neglecting any contribution from tissues beyond depth D, and by assuming "translational symmetry" to hold in the x-direction, Eq. (1) can be written as

$$v(\omega,x) = \int_{-D}^{0} \int_{-A/2}^{A/2} W(\omega,x-x',z')\, T(x',z')\, dx'\, dz' \tag{2}$$

Note that the translation simmetry implies that the biological body is stratified along the z-direction, i.e., its dielectric constant has no variation along the transverse direction within the explored region.

It is interesting to study two particular situations. When the unknown temperature is a function of $x$ only, Eq. (2) becomes

$$v(\omega,x) = \int_{-A/2}^{A/2} H(\omega,x-x')\, T(x')\, dx' \tag{3}$$

where

$$H(\omega,x-x') = \int_{-D}^{0} W(\omega,x-x',z')\, dz' \tag{4}$$

The inversion of Eq. (3) is called deconvolution and has been studied in many different physical contexts. It has been shown that superresolution, i.e., a resolution better than physical dimensions of used sensors, can be obtained by suitable processing [7]. Another particular situation is encountered when the unknown temperature is a function of $z$ only. In this case

$$v(\omega) = \int_{-D}^{0} G(\omega,z')\, T(z')\, dz' \tag{5}$$

where

$$G(\omega,z') = \int_{-A/2}^{A/2} W(\omega,x'-x,z')\, dx' \tag{6}$$

provided the integral in (6) is independent of $x$. Eq. (5) has been extensively studied in connection with the inversion of microwave radiometric data obtained by plane-wave observation of one-dimensional models of biological structures [4].

We now give a formulation which can be used when $T$ varies along both directions. In this case

$$W(x,z) = \sum_{n=-\infty}^{+\infty} w_n(z)\, e^{jn\frac{2\pi}{A}x} \tag{7}$$

where $A$ is the width of the region to be explored (Fig. 1). An analogous expansion is used for $T$:

$$T(x,z) = \sum_{m=-\infty}^{+\infty} t_m(z)\, e^{jm\frac{2\pi}{A}x} \quad . \tag{8}$$

By inserting Eqs. (7) and (8) into (2), the following expression of the radiometric signal is obtained:

$$v(\omega,k\Delta) = A \sum_{n=-\infty}^{+\infty} e^{jn\frac{2\pi}{A}k\Delta} \int_{-D}^{0} w_n(z)\, t_n(z)\, dz \tag{9}$$

where $v(\omega,k\Delta)$ is the signal received when the applicator is moved in the transverse direction by $k\Delta$ $(k=0,\pm1,...)$ to simulate the discontinuous displacement of the probe in actual measurements. It can be recognized that the integrals

$$V_n = A \int_{-D}^{0} w_n(z)\, t_n(z)\, dz \tag{10}$$

are the spatial spectral components of the measured $v(\omega,x)$. The unknown temperature distribution can be determined by (8) once its spectral components $t_n(z)$ have been retrieved. The retrieval of the individual $t_n$ $(n=0,\pm1,...)$ can be accomplished by solving the corresponding Freholm integral equation of the first kind (10) for which the needed set of data is obtained by a discrete spatial Fourier transform of $v(\omega,k\Delta)$ measured at different frequencies by a multispectral radiometer.

## RESULTS AND DISCUSSION

A numerical analysis has been carried out to test the performance of the method in reconstructing the temperature field in biological tissues. The impulse response of the reconstruction algorithm has been determined for a three-layer (skin-fat-muscle) plane parallel model of tissues. The reconstruction uses simulated radiometric measurements obtained by Eq. (1) for 32 positions of a rectangular waveguide receiving device and six frequencies 1 GHz apart up to 6.5 GHz. The input data refer to a Dirac pulse of temperature assumed at different locations in the muscle. The weighting functions W have been calculated by reciprocity for the fundamental mode of the applicator [8]. They are represented in a gray scale in Fig. 2 for the 1.5 and 6.5 GHz frequencies.

As already said, this kind of ill-conditioned inverse problem is characterized by instabilities of the retrievals produced both by errors in the data and by computational numerical effects. A constrained linear inversion [9] has been used to obtain acceptably stable results. Fig. 3 reports the temperature retrievals in the muscle obtained by the mentioned technique in the limiting case of no noise in the measured data. A low value ($10^{-8}$) of the smoothing parameter $\gamma$ was sufficient to quench the numerical instabilities in this ideal case. But where a 1% noise in the data was assumed, an increase of the smoothing parameter was needed to reduce the consequent erraticity of the retrievals. Fig. 4 reports the reconstructions obtained with a smoothing parameter $\gamma$ equal to $10^{-3}$. A general loss of resolution of the algorithm is observed, particularly for increasing depths in the muscle. Such a value of $\gamma$ has been used also in the case of a 10% measurement noise, with the results shown in Fig. 5. The reconstructions suffer from more instability, although the imposed constraint is able to counteract the effect of ill-conditioning at least in some regions of the tissues. The obtained results are believed to be encouraging towards the feasibility of multispectral microwave thermography, though experimental results must be awaited before definite conclusion on this matter be drawn.

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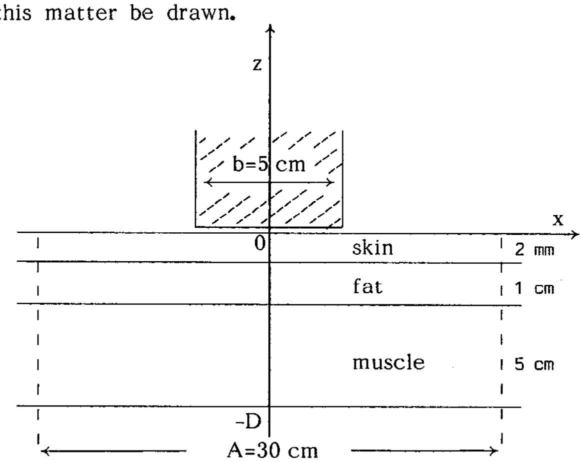


Fig. 1 Three-layer emitting medium (biological body) and waveguide contacting applicator. Dimensions used in numerical simulations are quoted.

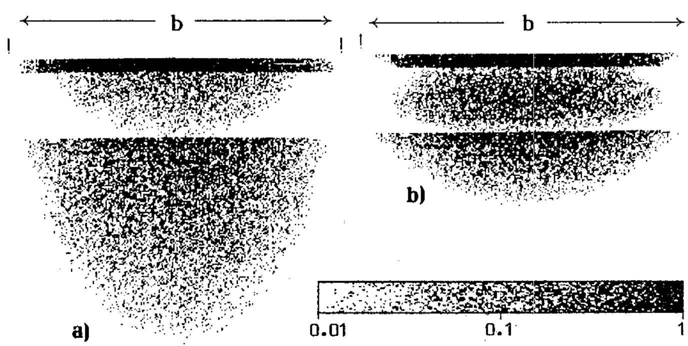


Fig. 2 Two-dimensional weighting functions for a rectangular waveguide contacting applicator (fundamental mode) in logarithmic gray scale for a) 1.5 GHz and b) 6.5 GHz.

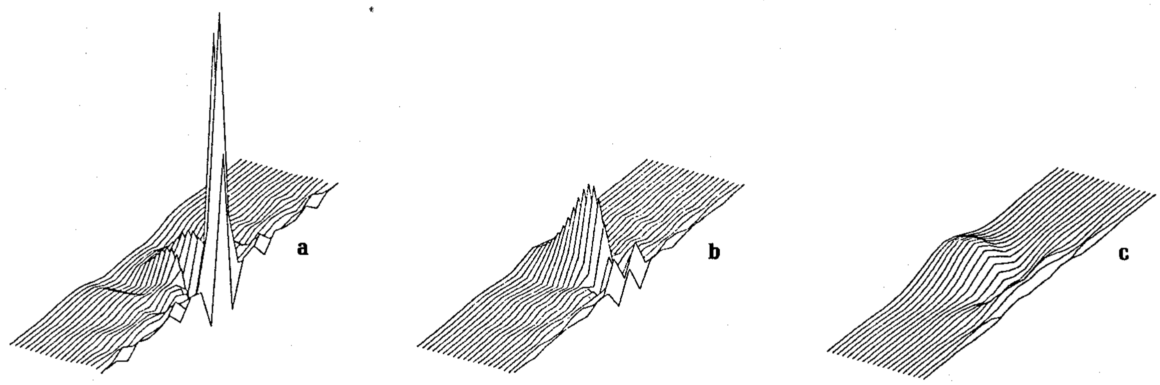


Fig. 3 Two-dimensional reconstructions of a temperature Dirac pulse centrally located (x=0) in the muscle and at variable depths: a) z=-1.5 cm.; b) z=-3.5 cm.; c) z=-5.5 cm. Noiseless synthetic data, 32 measuring points 1 cm. apart, smoothing parameter $\gamma=10^{-8}$.

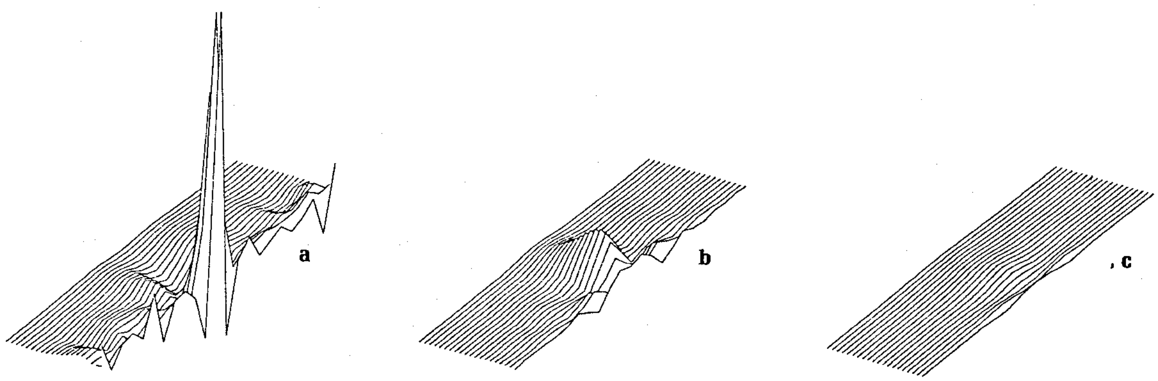


Fig. 4 As in Fig. 3, except that a 1% noise is added to the synthetic measurement data and the smoothing parameter $\gamma$ is now $10^{-3}$.

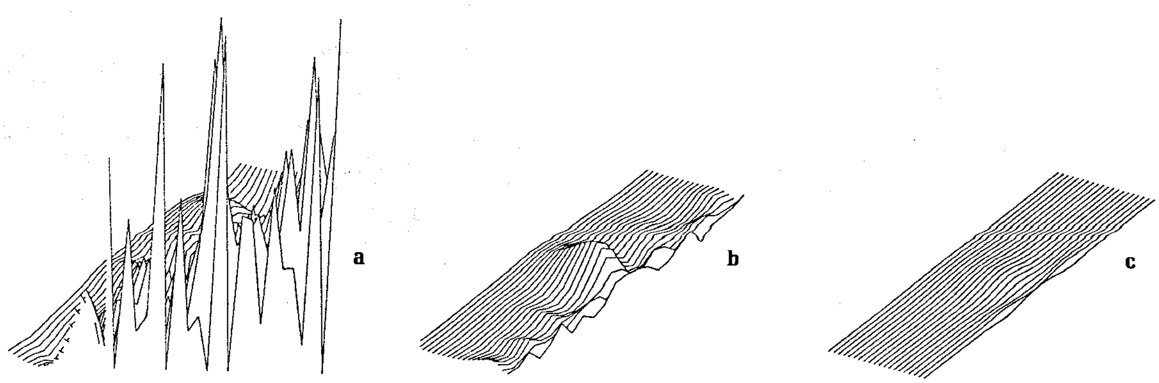


Fig. 5 As in Fig. 3, except that a 10% noise is added to the synthetic measurement data and $\gamma=10^{-3}$.